Integrated Calculators: Moving Calculation into the World

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ABSTRACT

Computing devices commonly act as tools, extending our abilities and shaping how we interact with the world. We investigate one such tool, the calculator, which helps with arithmetic, but also commonly offers specialized functions for conversions, formulas, or graphing. Through an analysis of calculator apps and use cases, we describe limitations of current calculators. Crucially, calculator apps remain detached from tasks, motivating us to explore how to more closely integrate calculation with the world through augmented reality (AR). AR calculators can directly use measurements and numbers from the world in calculations as well as display results of calculations in the world. We provide a conceptual account of calculation in AR, as well as video prototypes that concretize the concept across different scenarios. These examples demonstrate how moving tools like the calculator to AR offers tighter task integration and reduces the work required in translating between the world and computational tools.

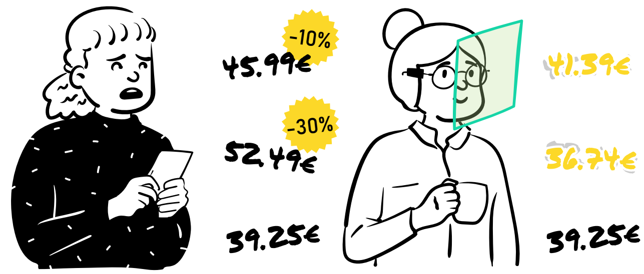

Figure 1: Calculators help in everyday situations, such as with discounts. Yet the left user is struggling to compare prices as she has to copy all numbers to her calculator, figure out how to apply the discounts, and then remember the final prices. The AR user on the right can instead use the numbers from the world directly in her calculation and see the results superimposed back in the world for easy comparison.

CCS CONCEPTS

• **Human-centered computing** → **Mixed / augmented reality**; **Mobile computing**; *HCI theory, concepts and models.*

KEYWORDS

mobile interaction, calculator, tool use, augmented reality, video prototyping, concept

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1 INTRODUCTION

Tool use extends our abilities and empowers us to tackle otherwise impossible tasks. They can integrate into our body schemas and using a tool can be as natural as acting with only our hands. Yet, with computational tools this kind of integration is not en-par with their physical counterparts. Instead, computational tools generally exist separate from the physical space and it is the user's job to translate to and from the tool. For example, while one can write a shopping list on ones phone, the actual products sit on shelves and users have to connect the two on their own.

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We focus on the calculator, a fundamental computational tool, and how it could be adapted to increase integration with tasks. Calculators have been around for millennia (e.g., the abacus) to support us in hard tasks, such as multiplying large numbers. Today, calculators are fairly standardized with familiar interfaces in their physical and app forms. However, this state of the art has also been put into question, such as by Thimbleby, who argues that these calculators can be confusing and not well aligned with users' conceptual models [52]. In this paper we explore the idea that calculators could alleviate some of this mismatch, by better integrating with the world they are applied in. We posit that augmented reality (AR) could bring about this better integration through novel calculator designs that exist within the world, instead of separate from it. Previous work has already shown that AR visualizations aid learning of mathematics, such as Kang et al.'s *ARMath* project [23]. In their review of AR use in education, Bulut and Ferri [6] also point to visualization of mathematical concepts as a key benefit of such systems, in addition to them being engaging and motivating.

Starting from an investigation of calculator use, we explore the concept of AR calculators by applying a video prototyping methodology. We explored multiple concrete scenarios and suitable interactions for them and illustrate how tasks and calculations could be integrated more closely than with current calculator apps. Building on that, we then distill these into a general conceptual account of an AR calculator that: (1) acquires inputs directly from the world, (2) enables arithmetic expressions on these inputs, and (3) projects the results of calculation back into the world. Note that we use the term calculation for the core process of mathematical operations, but also the overall task, including input acquisition and interpretation of the results.

Through our video prototypes and concept we aim to outline the potential for AR to better integrate tools with tasks. Implementation of this envisioned integration is an open challenge and we hence also discuss this challenge as well as how our concepts fits in with other work in AR and HCI. In summary, we contribute:

- a survey on the use of phones as calculators
- an analysis of current calculator apps
- several example designs of AR calculators
- a concept for how AR can be used to support tasks that require calculation

2 RELATED WORK

We focus on the calculator as a tool in everyday life and hence also everyday tasks, which relates to the notion of "everyday mathematics". We also point to technical works on improving interaction with calculators as well as integration of calculation with the world. Finally, our exploration of an AR calculator concept connects to previous work on using AR for everyday tasks.

2.1 Everyday Mathematics

Use of mathematics in everyday life has commonly been described from an educational perspective and in opposition to "school mathematics". Saxe [48] as well as Nunes et al. [44], for example, described the practices of Brazilian child street vendors and how "street mathematics" methods differ substantially from school ones. Squaring these perspectives can be challenging [12]. Another example of everyday math practices are Masingila's observations of workers in a carpet-laying business [33]. Their use of mathematical concepts included estimating how much material would be needed for a floor, figuring out how to cut material to fit around pipes, or translating between different scales. There are diverse uses of mathematics in everyday life [17], as also described in the 'Adult Math Project" [27]. For example, in grocery shopping people simplify and transform package sizes and prices in order to compare products more easily. The literature suggests that mathematics in the wild is quite different from school mathematics and, importantly for this paper, different from what is presupposed in most calculator designs.

2.2 Interaction Techniques for Calculators

A few papers have explored alternative means for interaction with calculators. *GestureCalc*, for example, enables number entry via taps and swipes to enable eyes-free use [10]. A similar approach was also explored with *DigiTaps* [1]. Another option is to use voice for calculator input and speech for output [5]. These interaction techniques offer particular benefits to users with visual impairments. Another case is Cairns et al.'s declarative calculator [9] which was an attempt to, by changing how commands are entered, make calculation easier. Overall, there has been a focus on input but little support for figuring out what to input; none of the papers are about projecting results of calculations back into the world.

2.3 Calculation in the World

Wellner's *DigitalDesk* showed how a virtual calculator can be overlaid onto a physical desk [59]. Numbers can then be selected directly from documents instead of needing to be typed in. With *AR DeepCalorieCam V2*, the calories of food can be calculated [51]. Instead of manual entry of food type and amount, this system directly recognizes food and estimates its size. The physical *SPATA* tools relay measurements from the real world to design applications [57]. User can then, for example, scale a virtual model to fit into a physical gap. These works enable partial forms of integrated calculation, primarily by allowing input from the world. Building on this work, we investigate how to integrate all stages of calculation in a general-purpose manner.

2.4 AR for Everyday Tasks

AR support for everyday tasks can come in the form of guidance, such as in assembly and fabrication tasks [3, 46, 50] or for moving instructional material into the world [37]. Another area is navigation where AR can help users moving through outdoor [21] and indoor [47] spaces. Bonanni et al. [4] as well as Chi et al. [11] looked at AR support in the kitchen with visual overlays for visualizing fridge contents, water temperature, and nutritional information during food preparation. Finally, Bhatia et al. [2] demonstrated that comic effects in AR can enhance everyday activities, such as by making users feel like they are running faster. These papers all demonstrate the potential benefits of integrating AR with everyday activities. We apply this to the area of calculation and also present a generalizable concept of how to translate between the world and application support.

3 ON CALCULATORS

With our goal of better integrated calculators in mind, we first set out to understand the tasks they are used in. Previous work is scarce here and, while there is a large amount of research on the use of physical calculators in the classroom (e.g., [13, 36]), the same is not true for their use in everyday life. We therefore conducted two surveys to understand how calculator apps are used: (1) an online survey on users' experiences with calculator apps, and (2) a survey of the calculator apps available on Google Play. We specifically focused on calculator apps as these are the most common form of calculator in use today.

3.1 Mobile Calculation Survey

We used *Prolific*[1] to gather data on situations and challenges around using their phone as a calculator (mean completion time of 4 minutes) from 100 participants (age 18–62, 64 male, 35 female, 1 undisclosed, paid 0.40 GBP each). Calculator apps were used regularly[2] and in a wide range of tasks. Commonly reported use cases included bill splitting, shopping, homework, cooking, and work. Calculators here were used for conversion, comparison, to work with large or many numbers, and to calculate percentages. These answers also highlighted some of the limitations of mental calculation, which necessitate use of a calculator, and one participant explicitly mentioned to use a calculator app when "mental calculation was not accurate enough". Overall, the survey showed that calculator apps are an important tool in daily life that fills a number of different roles and purposes. Subsequently, this points to potential benefits of further improving how calculators can support these activities.

[1] https://www.prolific.co
[2] Reported as "a few times in a" day (12 times), week (46 times) or month (30 times).

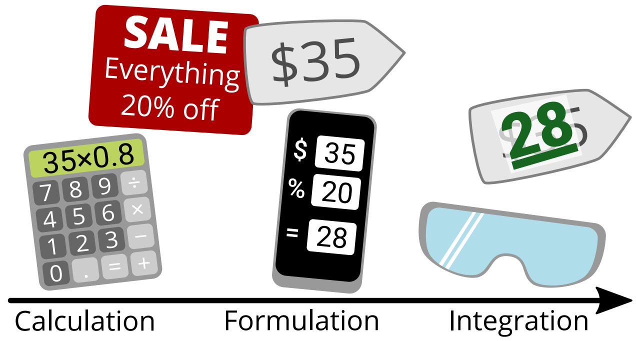

Figure 2: Apps have improved upon classic calculators by adding specialized views that alleviate the need to turn a task into an equation. AR calculators can further increase task support by integrating calculation directly with the world.

3.2 Calculator Apps Landscape

To supplement the personal accounts of calculator use, we conducted a broader investigation on what tasks current calculator apps support. We investigated apps found through (1) a 2016 dataset of all Android apps filtered to apps whose name contains the term *'calc'*, and (2) a Google Play search for apps that match a *'calculator'* query. We removed apps which were not available anymore, not calculators, or only provided the same basic functionality as the default one. This left us with 256 apps for analysis which we grouped into ten categories, such as *conversion*, *finance*, *shopping*, *tipping*, and *engineering*. Several apps offered dedicated modes for working with concepts like *times* (31), *fractions* (11), and *percentages* (6). For a full list of groups and more detailed descriptions, please see Section A in the appendix.

We identified four patterns across the calculator apps: (1) integration of other everyday tools with calculation, (2) tailoring to specialized use cases, (3) lowering mathematical barriers, and (4) striving to align the interface to the world and tasks. Please see Section B in the appendix for more details. In the context of improving task integration, the last two patterns are of particular importance. Together, they make it easier to translate mathematical problems into actions. Instead of requiring users to provide all inputs in the right format and sequence, the calculator provides a view tailored to the problem where users only need to fill in the gaps. For example, they do not need to know how to calculate mortgage rates, but simply enter numbers into appropriately labelled forms.

3.3 Summary

For the tasks mentioned in our first survey, the current support of providing abstractions and specialized views for input has clear limitations. Price comparison, for example, is supported in calculator apps with unit conversion, discount calculation, and summation. Yet, even with such views, users still need to enter sizes, prices, and discounts into the respective fields. Furthermore, the objects of interest are products that exist in the real world, but the comparison occurs on a screen, not in the world. We believe there is another level of task support possible, which focuses on *integration*. Instead of translating from the world (e.g., products in a store) to the calculator, the calculator then integrates with the world directly.

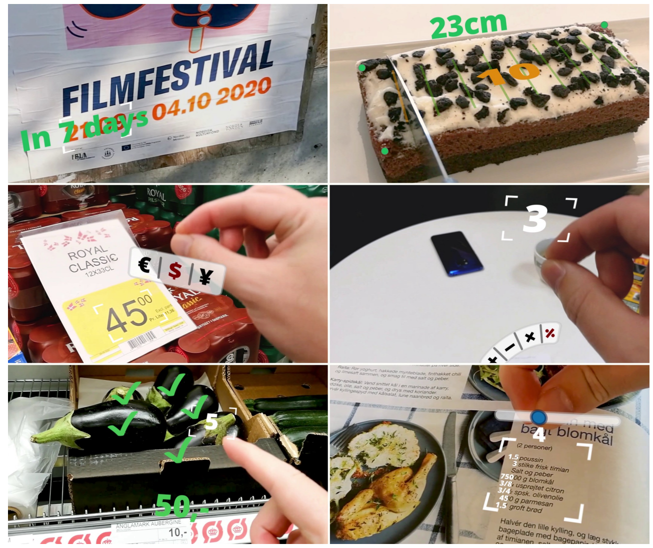

Figure 3: We created video prototypes of AR calculators for several situations. Clockwise from top left: (1) calculation with a date from a poster, (2) guiding cake cutting divider lines, (3) splitting a bill, (4) adjusting ingredient amounts in a recipe, (5) calculating the total cost of a purchase, and (6) converting prices to another currency.

4 VIDEO PROTOTYPING AR CALCULATORS

In order to develop the idea of integrated AR calculators, we applied a video prototyping method to explore potential approaches. Similar to "virtual video prototypes" [20], we combined real video footage with virtual content. We used the video prototypes for our process of experimentation and reflection around integrated calculation, but also to communicate the envisioned use. As described by Kinsley [25], such vision videos can have a "performative capacity" as well as "discursive traits", and along those lines, Wong and Mulligan have analyzed AR concept videos in particular [61]. As Vertelney [55] pointed out, video prototypes are "especially useful when designing interfaces for technologies that do not yet exist"; this is particularly true for the AR calculator design space. Video prototypes are also a form of speculative design [60], that we use to probe the AR calculator space as well as a form of exploratory prototyping, as described by Zamfirescu-Pereira et al. [62]. Finally, as pointed out by Halskov and Lundqvist [19], such an approach is useful for "filtering the design space".

There has been a range of research that uses AR video prototypes or presents new tools to create them. An early example of the former is Mackay et al.'s work on AR air traffic control interfaces [32] and more recently Lu and Bowman [31] used video prototypes to explore the concept of glanceable AR interfaces. Pronto [30] and 360proto [39] are examples of prototyping tools specifically for AR interfaces, both enabling use of sketching for that purpose. The Montage video prototyping system also was shown to support development of AR video prototypes [29]. Finally, Buruk and Hamari [7] have described how paper prototyping and video sketching can be used to create "immersive video prototypes".

In our video prototyping process we investigated several concrete tasks where calculation could be used in everyday life. Our choice was informed by our initial survey and app analysis, but also by what kinds of tasks could benefit most from closer integration. The tasks we picked are: (1) converting prices, (2) calculating totals, (3) cutting cake, (4) calculation with dates, (5) splitting a bill, and (6) adjusting ingredient amounts.

For each task, we recorded footage in the field as we engaged in these everyday activities. We filmed in several rounds on the street, in supermarkets, kitchens, and living rooms. After a shot, we imported the footage into Adobe After Effects, applied video stabilization, and ran its 3D camera tracker, which then allowed us to add virtual elements to the videos. In the process, we tested different input gestures and AR visualizations, ultimately arriving at a set of interactions based on crossing and pinching.

Figure 3 shows frames from each of the tasks in our final video prototype. The full video prototype can be found in the supplemental material. For each of the explored tasks, the design considerations and final choices are described below.

4.1 Converting Prices

As calculation commonly concerns prices, we designed an example of how AR can support this. Prices already exist in the world as printed numbers, as do modifiers for them, such as discount signs or sales banners. Calculation with prices can be direct, where users interact with the price tags, or indirect, where they interact with the products that are annotated by the price tags. Here, we illustrate an instance of the former approach.

In our example, the AR calculator recognizes price tags in the world and enables interactivity of their elements. Users can then "pick up" the price with their fingers to activate a menu of potential operations. Here, a selection of other currencies is shown and crossing through an item in that menu selects it for conversion. Upon releasing their pinch gesture, the converted price is calculated and the price tag is updated with an overlay to show it.

4.2 Calculating Totals

Another use of price tags is to determine the total cost of a purchase, such as when buying several apples. In this case, the price information needs to be combined with information on how much of a product is being bought. With an AR calculator, the price can be directly used in a multiplication. However, this requires that the amount of products to buy is also available to the calculator. One way to do this is to *count out* the items to purchase.

In our example, the user points at all the aubergines he wants to buy, in order to count them. This puts a mark on each as it is pointed to, but also creates an AR overlay with a running tally next to the aubergines. The selected aubergines can then become an input for calculation and the user again selects them by pinching on them. There then appears another variant of a crossing menu that offers users the choice what arithmetic operation to perform on the selected number. Dragging from the count, through the multiplication sign, and on to the price tag results in calculation of the total price for the picked aubergines and an overlay that displays that total next to the tag.

4.3 Dividing a Cake

A recurring task is splitting up of one or a collection of objects. For example, dividing up the fruits from apple picking, splitting the money from a garage sale, or sharing a pizza. We picked cake cutting as the instance of this task to illustrate, where a whole cake is split up so everyone gets a piece of equal size.

Support for cake splitting can be done in two ways: (1) dividing the cake into a desired number of pieces, or (2) interactive cutting support based on the knife positioning. In either case, the calculated number of pieces and cutting information is displayed on the cake itself. Here, we describe the latter approach, where the calculation is continuously updated based on the user's actions.

Bringing a knife close to the cake activates the AR calculator, resulting in an overlay that shows the bounds and size of the cake. By hovering the knife over the cake the user can then explore potential divisions of the cake. For example, cutting a small first piece would give a larger number of final pieces than cutting a bigger first piece. Once the user starts cutting, that "locks in" the given cake division. The segment marks then stay in place, guiding the user in where to cut to evenly split up the cake.

4.4 Math with Dates

Dates are another form of numbers commonly encountered in everyday life. For example, flyers or posters for events show the respective date, food items show expiration dates, and conference websites contain submission deadlines. Calculation for dates is often subtraction: how many days are there between a date and today? This can be hard for people (given that months differ in length) and hence calculator support for this task is worthwhile. Here, we illustrate how the first use case, working with a date on a poster, can be supported by AR calculators.

Users are able to "pick up" the date from a poster for an upcoming event. This brings up a menu of arithmetic operations to perform, where the user crosses through the subtraction symbol to select that operation. The user could now end the action on top of another date to calculate the difference between the two. However, in this case we want to calculate the difference to the current date. Because the initial number was a date, the AR calculator also provides an abstract item for "today" that can be used just like a number from the world. By dropping the selected date on that item, the difference is calculated and then shown on top of the date.

4.5 Bill Splitting

Splitting a bill can be complicated and hence there are many apps for this. Using an AR calculator alleviates the need for people to take out their phone after dinner and instead lets them do the bill splitting directly in the world. We explore a basic example: splitting a bill evenly between several people.

Splitting a bill requires the bill amount as well as the number of people to split by. We use a counting action again, creating a tally of how many people are sitting in front of the user. The user can then pick up the total from the bill, select division from the menu, and end their action on the tally. This divides the price by the number of people and hence gives the individual contributions. In the case of a bill, the resulting amount is then displayed next to each of the people that made up the tally in the first place.

4.6 Adjusting Recipe Ingredients

Finally, we address cooking, specifically adjusting a recipe to the amount of people that are coming for dinner. In this scenario, the user bought a magazine with a good recipe, but that recipe is only for two. As they expect nine people to come to their dinner party, they need to figure out the adjustment factor and multiply each ingredient amount by that factor individually.

Instead, the AR calculator recognized the recipe and its numeric elements (i.e., ingredient and portion data) become available for calculation. The user can then "pick up" the indicated amount of diners to manipulate it. Here, this action brings up a slider, enabling the user to select how many people they want to cook for. As they change that number, the needed amount of ingredients automatically updates and is shown next to the original ones.

5 A CONCEPT FOR MOVING CALCULATION INTO THE WORLD

Through our video prototypes we have developed an understanding of actions and effects we would expect to see in AR calculators. Here, we abstract from these concrete examples and outline a general concept for integrated calculators. This integration is achieved by removing the need to translate between the world and the calculator. Integration hence requires the ability to *sense* the world in front of the user as well as the ability to layer output on top of that world. Most commonly, this means visual perception as well as graphical output, but can also include other sensors and feedback modalities. While this is partly possible with phones, AR glasses offer the ideal combination of capabilities for this kind of integration. Current AR glasses, such as *Snapchat's Spectacles* or the *Vuzix Blade*[3], already provide some of the world sensing and augmentation capabilities we assume here. We posit that, with growing maturity of AR glasses, they will be able to support the integrated calculation described in the following.

As illustrated in Figure 4, integration of calculation into the world has three components: (1) **acquisition**, where parts of the world are converted into a numerical representation, (2) **operation**, where calculations are performed on one or combinations of these representations, and (3) **projection**, where the results of these operations are shown back in the world. Together, they enable all kinds of calculations, supporting the range of everyday tasks described earlier. These three components also define a *calculation flow*, instead of a single calculation, as the world is not static and thus calculation should not be either. For example, as a balloon deflates, a calculation of its volume should reevaluate and updated results projected onto the world. The three components also do not necessitate a sequential process and integrated calculation can be iterative. For example, users might want to modify the acquisition step in response to a shown result.

As an example of this concept, consider the scenario in Figure 4: cutting a piece of wood into segments of equal length. Here, the user (1) selects the work piece and then (2) picks its length property to use in calculation. As the task is to cut that piece of wood into a number of segments, the user then (3) picks division as operation and (4) picks a divisor, which in this example is the number four.

[3]spectacles.com and vuzix.com/Products/Blade-Smart-Glasses respectively.

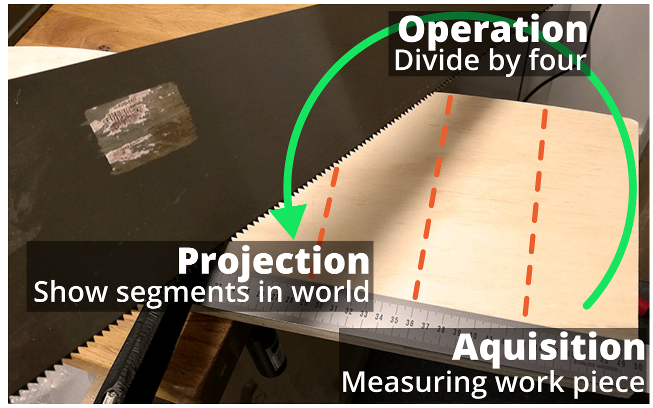

Figure 4: AR calculators have three components: (1) they acquire numerical data from the world through text recognition or measurement, (2) they perform operations on that data, and (3) they project the results back into the world. In the shown example, the target object for calculation is a piece of wood that is cut into segments.

An AR calculator can then measure the piece of wood and divide the length by the desired number of splits to determine how long each segment needs to be. As calculation is integrated with the world, the calculator then (5) shows the resulting segments and where to cut directly on the piece of wood. Next, we discuss each of the components individually with respect to their requirements and design considerations.

5.1 Acquisition

In current calculators, acquisition is the sole responsibility of the user, yet AR calculators offer extended support. We describe as acquisition any method that makes parts of the world available as input to calculation. We distinguish two kinds of acquisition: (1) text-based and (2) measurement-based.

5.1.1 Text-Based Acquisition. Many things relevant to calculation come in the form of text, such as on price tags, receipts, furniture catalogs, and cookbooks. In addition to extracting numbers directly, they can also be inferred from context, such as when recognizing bank notes [56]. Numerical data in text format often comes with unit information, which can be explicit (e.g., units in a recipe), or implicit (e.g., currency information not printed on a price tag).

Apps already exist to extract and process the contained data from some documents (e.g., for invoices and receipts). Similarly, AR devices could include OCR to detect numerical data and provide users a way to use it. Users might, for example, want to sum up calorie numbers from food packaging, calculate taxes from price tags, or calculate the number of days till a date shown on a flyer.

5.1.2 Measurement-Based Acquisition. Measuring the world is another way input for calculation can be acquired. A basic form of this is *counting*. For example, one could want to count the apples on a table, the coins in a wallet, or the number of cars passing through an intersection. Some counting, hence, is of things available concurrently, other counting takes place over a period of time.

There are a number of different counting techniques, such as verbal or finger counting [15] and users keep track of the running tally in different ways (e.g., making a mark). Correspondingly, integration of counting can happen in several ways: (1) by taking over the tallying, (2) by visually "crossing out" what has been counted, and (3) by "autocompleting" users' counting actions. In any case, counting results in acquiring the relevant number and representing it in the world, for instance next to the objects that were counted.

Tallying support frees users from having to keep track of the tally themselves. Instead, users only need to tell a system when to increase the tally by, for example, pointing to an object or making an utterance. A "crossing out" of already counted objects can further support users by decreasing the risk of double counting. Finally, systems could detect what a user is counting and extrapolate their actions. For example, when counting items on a table by pointing at them, it would be sensible to assume a user will want to count all of them. Refinements of this are possible, such as recognizing target object classes or grouping counts by object characteristics.

Not everything is countable and hence *other forms of measurement* are needed. For example, flour and water are uncountable objects, yet can still be measured in the form of weight or volume. Similarly, while one can count the number of couches, the size of an individual couch needs to be measured, not counted. Other measures are not based on individual objects, but rather on parts of the world, such as the size of a wall, or the brightness of a room. Furthermore, dynamic properties of objects can also be measured, such as the speed of a car or the loudness of a shout. Just as with text-based acquisition, measurements also carry unit information, depending on what property is measured.

For measurement of the world there already is a rich set of existing apps, demonstrating feasibility. Especially measuring of spaces (e.g., to build floorplans) is something many apps address. But there also are a large number of generic "ruler" apps (e.g., Google's *Measure* app) that use the AR capabilities of phones to replicate the functionality of physical rulers. In addition to lengths and areas devices could, dependent on the included sensors, also measure quantities such as speed, brightness, loudness, or tilt.

Finally, some things cannot be accurately counted or measured, but might still be used for input through *approximation*. Estimation skills are an important part of mathematics and play a role in much of its everyday use [35]. Similar to how AR devices can measure things in the world, they can also be used to get approximations. An example of this is calorie estimation of food using deep learning [51]. Approaches leveraging machine learning have also been shown to help in estimating people's age [40] and house prices [28]. This demonstrates the potential for training models that are able to make estimates based on patterns not necessarily obvious to people. Subsequently, such approximations can then be used in calculation to, for example, sum up the calories from the week, or include building valuations in a mortgage estimate.

5.2 Operation

After acquisition of data from the world, calculation proceeds through operations on this data. Which operations are possible and appropriate then depends on what kind of data was acquired in the first place. For example, currency conversion is only a sensible operation on prices and similar pieces of data. Similarly, addition only makes sense where the things being added are compatible (e.g., the result of adding five apples and three people is ill-defined).

We can distinguish between different classes of operations. Some operate on singular inputs (e.g., converting a price), while others require multiple inputs (e.g., division). Operations can also supplement input from the world with numbers entered directly (e.g., to multiply with a known factor). In general, operations are user-controlled, but in some situations automatic operations are useful too (e.g., overlaying currency conversions during a trip). In either case, picking of operations can benefit from contextual information (e.g., limiting conversion to certain kinds of objects and units).

5.2.1 Operations on Singular Inputs. Singular inputs are things like a scanned price tag, a stack of apples, or a measured piece of wood. Several operations are possible using just those inputs on their own. Most prominently, this is conversion where one might want the price in a different currency or a distance in a different unit. But conversion can also trigger another acquisition step, for example, when after selecting a few apples one gets their weight estimated for subsequent use.

When operations are performed on numbers that exist in the world as text, such operations can use direct manipulation. For example, upon detection of a tag showing a price of "$3.99", users could trigger a currency conversion by manipulating the "$" and changing it into a "€". Similarly, numbers on an input could be manipulated to, for example, determine how many packages to buy for a desired number of servings, or how much of a paint can would cover a certain number of square meters. In general, direct manipulation is an option if a change in one number or unit has a meaningful effect on other parts of the world. Instead of entering a factor, divider, or picking a conversion, these operations are then implicit in the enacted change.

Where no direct manipulation is possible, contextual information could be used to offer sensible operations on a given input. For example, one commonly needs to half, double, or split in three and hence these options could be available directly. But operations on singular input may also be *relative to* a baseline or context-dependent standard. For instance with dates we are often interested in their relation to the current day (e.g., in answering questions like "in how many days do we go on holiday"). Similarly, if we inspect a number that denotes the amount of paint left in a bucket, we could be interested in that amount relative to the original amount.

Finally, users sometimes will want to store inputs away for later use. This could be necessary because the calculation they want to perform has inputs that are far away from each other, or exist at different points in time. A "storage" operation on an acquired input hence retains it in the calculator and makes it available for multi-input operations, as described below.

5.2.2 Operations on Multiple Inputs. With multiple inputs, arithmetic operations can be performed between them. However, not every combination of measures is semantically or syntactically meaningful. For example, it makes little sense to multiply the amount of flour in a recipe by the surface area of a roof. Yet, multiplying the same by a number of people would be a reasonable combination. To constrain what can be operated on together, contextual as well as semantic information should hence be used.

In any case, users will specify which parts of the world they would like to use in an operation. With direct manipulation, they might drag from one to the other to make this choice. They could also use inputs they have previously stored away (i.e., as symbols within the AR calculator). But inputs could also be added automatically based on contextual information (e.g., in a restaurant it might be sensible to automatically make the number of people at the table available for calculation).

Users could pick operations either before or after the inputs. Choosing the operation in-between the inputs is a good choice if there are few options. Alternatively, users first select the operands of a multi-input operation and are then shown a list of possible operations. Where the space of potential operations is large, filtering it down to only those possible given a set of inputs is important to keep such a system usable.

5.3 Projection

For better integration of calculation, just as the inputs come from the world, the results should be shown in the world as well. In contrast to classic calculators, this allows for results to be shown in a format that aligns with the initially posed problem. Instead of unitless numbers on a screen, display then can be through a wider range of visualizations, depending on the given calculation.

There are many different ways to visualize information in AR [63]. For the purposes of our concept, we primarily distinguish two kinds of visualization: (1) on-object, and (2) around-object. Furthermore, we note that there are differences in the *content* that is shown in these ways. For many calculations a resulting amount, ratio, or transformation of the original input is shown, such as converted or summed up prices. However, in some scenarios the result of a calculation is to be immediately used for a closely aligned action in the world. For example, consider a carpenter calculating how to split a 2×4 or a painter mixing colors. We describe considerations for such *action guidance* separately.

5.3.1 On-Object Augmentation. The result of some operations can be shown directly on the objects that were used as input. For example, the result of a division can be how to split up an object, such as a piece of wood or cake. Similarly, subtracting a length from the same piece would also result in a line showing the resulting length. Results of subtraction or division with groups of objects can also be shown by highlighting the "remaining" ones.

Depending on the objects and operations in question, different kinds of visualization are possible. Split lines or volumes are a suitable method when working on single pieces (e.g., the aforementioned piece of wood). Coloring or other markings of resulting parts is more flexible and also works well with multiple inputs (e.g., highlighting the coins from a pile that are part of the result).

When textual data from the world was used as input, the result of calculation can be shown on top of the original. For example, when converting a price, the new price and currency can be shown directly on the tag. Similarly, ingredient information in recipes, dates on flyers, or distances on a sign can be directly replaced. Such kind of replacement is currently already in use by translation apps that overlay the translated text on the world.

5.3.2 Around-Object Augmentation. In some cases, the results of an operation cannot be shown directly on any of the inputs. For example, this is the case where the result is larger than the used inputs (e.g., after addition or multiplication). One solution there is to show the required additional amount next to the inputs (e.g., displaying two more virtual paint cans next to the real one). Alternatively, this can be done more indirectly by showing numerical factors next to the inputs (i.e., rendering a "3×" in the above example).

Marks and numbers next to the inputs are also a flexible alternative in other situations. For example, when counting objects the resulting number should be shown somewhere close to them. Dividing lines can be shown *between* individual objects to indicate how to split up a collection. Dashed lines could indicate by how much to extend a work piece to get to the desired length.

5.3.3 Action Guidance. Where some calculation is done for planning or evaluation purposes, other calculations are more deeply embedded within users' actions. Hence, the results of such calculations need to be directly actionable as well. For example, consider shortening a piece of wood or fitting trim around a corner. In both cases, measurement, calculation, as well as transfer of the result back to the workpiece are required before making the cut. AR calculators can help with each of these steps and ultimately provide direct guidance for the desired action. This guidance can be provided by, for example, marking where to cut or showing how to distribute. Fundamentally, if the result of a calculation is in reference to the world, that result should be shown in the world.

Action guidance can also be interactive, where a calculation updates as users act on an object. For example, holding a saw to a plank, a scoop to a bucket of ice cream, or a paint brush to a wall all signal an immediate action. AR calculation could then trigger based on a tool's presence or positioning to show results of the corresponding potential actions. If the volume of the mentioned ice cream bucket is known, for example, the number of scoops available could be displayed.

6 DISCUSSION

We have outlined a concept for moving calculation into the world using AR. This addresses the basic dilemma of calculators, be they physical or in applications: the disconnect between the world and the calculator. As a general purpose tool, calculators require translation of the activities and tasks of the real world into a format that can be entered into them. This can require users to count, measure, or know formulas. This is often difficult and hence this is why many apps are tailored to specific tasks. A similar difficulty exists in integrating the results of calculations back into the world. The calculator only shows a number, which has to be interpreted by the user back in the original context of the calculation. We have discussed how to use AR to (a) turn parts of the world into numbers, (b) operate on those numbers using direct manipulation and predictions of the appropriate calculations, and (c) project results into the world to give numbers form, sense, and context. Next, we discuss how to realize this concept with AR, how to develop it further, and how it relates to other work on HCI and AR.

6.1 Realizing AR Calculators

Physical calculators are general purpose; scientific calculators contain more specific tools; and the apps we analyzed cover both general-purpose calculators as well as specialized ones. Our preference for realizing the concept is as an application-agnostic service in AR devices (e.g., as an OS service). This preference is similar to Raskin's vision of unification [45]. He argued that "by liberating commands from applications, we eliminate the inherent modality of applications". Our intent is similar: liberate calculations from calculators and put them into the world. This requires some balance, though, because our surveys suggest that users want specialized apps. For those, we have presented context-aware rules and mechanisms for predicting specialized acquisition, operation, and projection that match tasks in the world. It is difficult to make that integration sufficiently application-agnostic so that all calculation would be supported in similar ways.

We have worked with AR-based calculation at the conceptual level. We have presented an overall model for thinking about the concept, identified key steps in translation between the world and calculation, and illustrated a series of different application that reflect the concept. Yet, realizing this concept in actual AR systems raises a number of technical questions. First and foremost, a high level of scene understanding is necessary to make AR calculators work. For example, they need to be able to efficiently and reliably detect and extract text and numbers. This requires sufficiently sensitive and high resolution cameras, but also substantial effort in software. The real world is messy and there is no unified style or format for price tags, flyers, recipes, or receipts.

Measuring the world and calculating with that data brings another set of challenges. As a starting point, robust object detection is needed for counting of objects or deriving other properties from them. For example, measuring the length of a piece of wood requires segmenting it first. Explicit user action can lessen some of those constraints, but also decreases the amount of automatic support an AR calculator can provide. With respect to output, the challenges are less daunting. Rendering numbers and text around and above objects is already possible with current AR technology. More closely connected annotation of split lines and other forms of on-object results, however, require higher precision information on the objects' geometry.

A general purpose AR calculator still requires substantial improvements in the above aspects. However, currently apps tailored to more specialized scenarios already demonstrate a subset of the envisioned functionality. For example, there are mobile AR apps available for scanning equations, counting, and measuring[4]. Yet, phone apps also preclude full integration with tasks, as there is at least one hand holding the phone.

Implementing calculators for AR headsets comes with further challenges. Compared to phones, current headsets are more limited in compute capabilities and offer less mature development environments. Furthermore, headset AR requires new means of input (e.g., via hand tracking [38]) that is more complex and error-prone than touch.

[4]Arcana Studio's Augmented Reality Calculator, the CountThings app, and the *Measure* apps from Apple and Google respectively

6.2 Bringing AR into the world

Our concept is informed by earlier work on AR for everyday tasks [4, 11]. We add to this space, by imagining a way of embedding calculations in the world to help people accomplish real-world tasks. At a more general level, research on AR has explored generic and domain-specific organization of information and interaction. For instance, Grasset et al. [18] described some of the considerations for redesigning a book in AR with respect to organization and interaction. The concept we propose adds to such lines of work by discussing general possibilities for integrating AR in the world. Kim et al. [24] noted that "a large number of papers was focused on novel perceptually-based user interfaces which were inspired by natural interaction but were not constrained by physical limitation". Their point is that those interactions that merely mimic existing technology are inherently restrictive by focusing on the natural. In contrast, our work has tried to reimagine calculation in the world.

6.3 Relation to Other Work

The concept we have presented draws on much earlier work in HCI; next, we discuss the overlap and differences. First, earlier work has focused on tangible user interfaces for augmented reality [18, 59]. Calculations in AR share with that work the ambition of using tangible objects for haptics and as anchors for interaction. Outside of AR, Wellner's DigitalDesk [59] Calculator, has a similar loop between the physical and digital. As with that work, the division of work between AR and the real world is difficult to design. Our model of acquisition, operation, and projection gives one suggestion on how to organize and switch between physical objects, augmented numbers, and operators.

Second, the driving motivation behind calculations in AR is to minimize the difficulty users face in executing and evaluating their progress in doing real-world calculations. This is similar to Norman's work on executing and evaluating interactions [41] and classic work on articulatory and semantic distance in direct manipulation [22]. The key difference to our work is that in AR calculations, the whole task structure of calculation is changed—the challenge faced by designers is much more akin to concerns about functional allocation [8]. This is because the fundamental concern is task organization, rather than the directness or ease with which users carry out a command.

Third, as mentioned our work is related to ambitions to the unification of interaction [45]. At the same time, we are sensitive to the concerns of Norman [42] about applications that "have far too much power for the use I make of them, yet lack all the necessary components for any given task." Similar concerns have been voiced in discussing the role of applications in computing [43]. Thus, the difficulty here is balancing a standardized way of supporting calculating that is sufficiently integrated in the task to be useful and remove issues related to getting numbers in and out of calculators.

Fourth, 25 years ago, Scaife and Rogers [49] argued that too little is known about how graphical representations work. They discussed distributed cognition and knowledge-in-the-world as concepts that help appreciate the work such representations do. Distributed cognition, however, is difficult to use directly for redesigning calculation, because designs include not just representations, but also division of tasks and application of operators.

Fifth and finally, the ambition of integrating calculation in the world face similar challenges to those encountered in ubiquitous computing and embodied interaction. As pointed out by Dourish [14], we are always acting in a material and social world.

6.4 Limitations and Future Work

The concept we have proposed needs to be validated empirically. On the one hand, that requires a technological instantiation. We outlined some of the steps to do so in an earlier section. On the other hand, the experience of calculation integrated in the world needs to be assessed. Earlier evidence suggests that engaging with physical objects improves problem-solving and thinking [26, 54], decreases workload, and reduces anxiety [53]. This literature could help inspire gestures and organization of the physical space for calculation. Future work should investigate if these expectations hold with mathematics integrated in the world using AR and has benefits for mathematical understanding.

Integration with computational tools has been presented as an overall direction for the field [16]. We similarly see integration into tasks as a way to increase the support these tools can provide to users. But it is also possible that such integration could have little or even detrimental effects on users. AR calculators applications will need to be more efficient and effective than existing apps as well as mental arithmetic or pen and paper. This will require user studies and direct comparison of different ways to perform calculation.

In our concept we focus on visual integration of calculation with the world and hand-tracking for input, yet another possibility is to use audio and speech instead. Current voice assistants are able to answer mathematical questions (e.g., *"what is three times twenty-one?"*) as well as conversions (e.g., *"how many liters are in a gallon?"*). There is potential to pursue a closer integration within this modality as well, yet this will require similar advances in scene understanding (e.g., to be able to answer questions like *"Which of those lawns is larger?"*). Furthermore, multi-modal calculators that integrate these two and possibly further approaches would be worthwhile to explore as well.

7 CONCLUSION

Calculators and calculation are ubiquitous and widely used in everyday life. Yet, the calculation tools used in such situations have received little attention. We have investigated calculator apps as well as surveyed how people use them. An important limitation of these existing calculators is how they are inherently dissociated from the problems that people are trying to solve in the world. Users have to translate the world into numerical information, figure out how to operate on it, and translate back from numerical results to an interpretation in the world. Based on this observation, we have proposed a concept for integrating calculation with the world using AR. This integration draws on the benefit people see in general calculators as well as specialized apps. We have shown, on a conceptual level, how acquiring numbers, operating on them, and projecting them back into the world can bridge the gap between the calculating tool and the world it is used in. We argue that this type of integrated calculation has the potential to lower syntactic and semantic errors in calculation, ease relating results to the world, and help users think.

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A OBSERVED CALCULATOR TYPES

As described in Section 3.2, we analyzed and tagged 256 Android calculator apps. In a first pass, we assigned tags to each app using open-ended coding. Afterwards, we iteratively refined the tags and ultimately grouped them into ten clusters. The majority (163) of apps were assigned only one tag, but many apps supported multiple functions. We had 58 apps that were assigned at least three tags, while the average app had 1.9 tags. Figure 5 shows an overview of the identified tags as well as which tags co-occur. Below, we provide details on each of the ten subsequently identified groups and the included apps.

A.1 Scientific Calculators

We tagged 60 apps as scientific calculators, relating to a class of physical calculators with more complex layouts and functionality. For apps, the distinction is more fuzzy, as many offer similar functionality in alternative button modes, when used in landscape mode, or on larger devices. Yet, several apps copy their layout directly from physical scientific calculators. Commonly, scientific calculator apps include graphing functions and of the 32 graphing apps, 26 were also scientific calculators. Similarly, all 26 calculator apps that included some kind of solver were also scientific calculators.

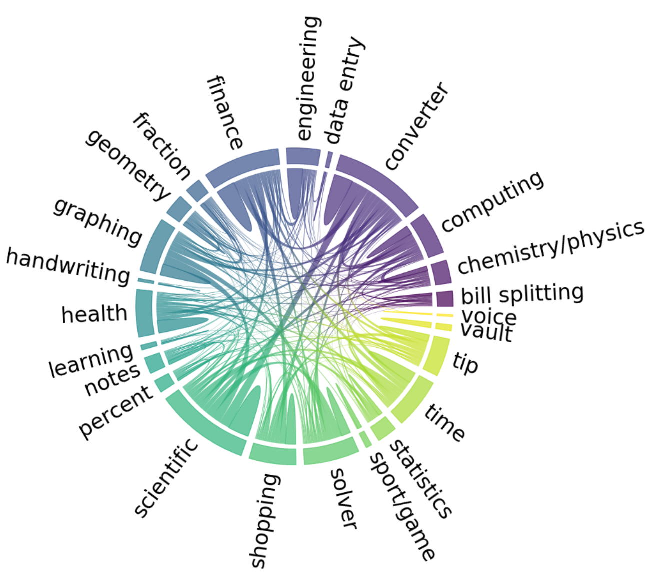

Figure 5: Occurrence and co-occurrence of functionality in the investigated set of Android apps.

A.2 Physics, Chemistry, Computing, and Engineering

We found 13 apps with functions relevant for physics or chemistry, 25 apps related to computing, as well as 43 apps tailored to engineering. A common function in these apps were collections of formulas and constants. Calculators for physics/chemistry also help calculating molecular weight, fertilizer use, and diluting and mixing solutions. For computing, functionality included conversion between binary, octal, hexadecimal, and decimal number systems. Furthermore, logical operations and shifting were commonly included and several apps also offered operations on network addresses. In engineering, we found a range of construction calculators that help with, for example, calculating how many bricks are needed for a wall, calculating the weight of a steel beam, or working with concrete or lumber. Apart from construction, workshop use (e.g., calculating bend allowances or blanking pressures) and electronics (e.g., Ohm's law or calculating circuit resistance) stood out. Other specialized calculators support hydraulic and pneumatic systems, surveying, photography, audio, lighting, and exhausts and heat transfer.

A.3 Conversion

Conversion was a function in 54 apps, most commonly in the form of currency conversion. In addition, a range of other units (e.g., weights, lengths, temperatures, energy, or speed) often could also be used in conversion. Some tasks of daily life are supported with dedicated converters, like ones for fuel consumption, cooking, as well as shoe, clothing, and jewellery sizes.

A.4 Health and Medical

We tagged 36 calculators as containing health or medical functionality, mostly apps including a body mass index calculator. This was sometimes accompanied with body fat percentage, basal metabolic rate, obesity index, and daily calories calculators to help with weight loss. Other functions were risk calculators for cardiovascular diseases and type 2 diabetes, several calculators related to pregnancy and menstruation, and one for calculating blood alcohol levels. In addition, there were professional medical calculators with functionality to, for example, help determine intubation tube sizes, or to compute sequential organ failure assessment scores.

A.5 Sports and Games

Only 12 apps contained functions specific to sports and games. The former included support for running, diving, weight lifting, shooting, and general fitness. The latter group had odds calculation for poker and a Pokemon Go calculator.

A.6 Financial

We tagged 54 apps as containing some form of financial calculation. This ranged from apps that calculate sales tax, to ones helping with loans. For example, there were functions to calculate loan amortization, time value of money, investment plans, mortgage payments, returns of investment, pip values, and capital asset pricing models.

A.7 Shopping and Dining

Related to financial calculations, some apps provided dedicated functionality for shopping (30), tipping (19), and bill splitting (6). Examples of shopping support we have already discussed were sales (and similar) tax calculation and size conversions. Additionally, many apps provided discount calculators and some integrate a shopping list functionality. Similar to discounts, tipping calculators allow users to work with initial costs and percentages. Bill splitting calculators also commonly take tipping into account when dividing up a bill, but some also provide control over rounding and allow for direct sharing of the calculated amount with friends.

A.8 Time

Calculating with time was a feature in 31 apps. This can be, for example, calculating the number of days between two dates, computing relative dates, setting countdowns, or converting between time representations and zones. Some apps allowed arbitrary math operations on times, such as multiplying and modulo. Functions around birthdays were common, such as computing time till a birthday, or age from a birthday.

A.9 Other Mathematics

For some areas of mathematics, calculators had special adaptations. For example, 12 apps had dedicated support for geometric operations, such as computing areas, volumes, intersections, or circumferences. We also found 11 apps with some kind of statistics functions. This included, descriptive statistics, statistical distributions, correlation, and regression.

All calculators can handle fractions and percentages, but some provide specialized support. We found 11 apps that stress their fraction functionality, including special display and conversion modes as well as input tailored to fractions. Similarly, 6 apps were designed for working with percentages beyond the tip and discount calculators discussed above. For example, such apps have dedicated screens for converting between percentages and fractions or to calculate the results of a percent operation.

A.10 Other Uses

A small number of calculators supported input via voice (3) and handwriting (2). There were also 4 calculators specifically designed for learning mathematics. Likely due to the inconspicuous nature of calculator apps there were 15 apps that, while also being a calculator, were primarily intended to hide files. Upon entering a secret code or performing a secret gesture, such 'vault' apps reveal a media gallery, private browser, or notes.

There were also 5 apps designed with some form of data entry in mind: counting money, recording weather observations, hours of work, or lumber. Some form of note taking was also supported in 10 apps. For example, while many more apps keep a calculation history, some of these allow users to add comments to entries in that history. Another calculator allowed users to freely mix text and number entry and operate in a style more like a notebook. Finally, some calculators had dedicated note or shopping list views directly in the app.

B CALCULATOR APP PATTERNS

We identified four patterns in the reported use and the analysis of available apps: (1) calculator apps commonly combine calculation functions with a multitude of loosely related additional everyday tools, (2) highly specialized uses manifest as tailored apps, (3) adaptations are available to lessen requirements of mathematical knowledge, and (4) functions are designed to align with the world and tasks within it. We describe each of these in more detail below and describe the resulting requirements for better calculation support.

B.1 Calculators between Multi-Tools and Specialization

Calculator apps contain a wide range of functionality (such as shown in Figure 6), but often include several non-calculator tools as well. Examples of such tools include spirit levels, compasses, flashlights, rulers, stop watches, and barcode readers. The calculator hence is sometimes just part of an assortment of tools, similar to a physical multi-tool. These strive to be handy in a range of everyday situations, underlining the role of the phone as an everyday tool [58, p. 56]. But it also shows that some see value in integrating all these tools into a single app, instead of having to move between apps for different tasks.

The challenge to users is that a concrete calculation task in the real world requires them to find the appropriate functionality. This is partly because function labels do not necessarily align with the given task, but also just because of the large number of functions available. Similar to the issue of bloatware in desktop computing [34], more functionality can negative impact usability.

B.2 Specialized Calculators

We found many areas, such as medicine, electronics, or forestry, for which specialized calculators exist. A common theme with these is that they provide presets for formulas relevant in their areas. For example, medical practitioners can calculate scores (e.g., HAS-BLED) without needing to memorize the respective formulas and parameters. In addition to relief from memorization, this also enables faster calculation. Instead of having to enter a full formula (and potentially transforming inputs to it), users only need to fill in the blanks. Specialized forms can also offer additional information on fields, such as parameter names and ranges, that are not available in generic calculators. Specialized calculators are commonly designed for experts and assume that these users are familiar with the domain. For example, such apps often use abbreviated units, codes, and symbols that users need to already know.

B.3 Easing Application of Mathematics

Many calculators have redundant functions, such as the common 'percent key' that multiplies the preceding number by 0.01. Similarly, some calculators provide extended support for working with fractions, such as comparing them. Some unit conversions (e.g., centimeters to meters) are also easy multiplications, yet not all users might be aware of this and hence they are dedicated functions. Furthermore, several calculators provide step-by-step results, which also can help with mathematical understanding.

Many calculator functions hide more complex math by providing simpler forms. For example, financial calculators provide abstractions for a range of comparably simple formulas, such as interest calculation. While the total debt can be calculated as $amount \cdot (1 + rate)^{time}$, this is likely a non-trivial transformation for many. Hence, many calculators contain specialized forms with fields for each of the required values, but hide the equation itself.

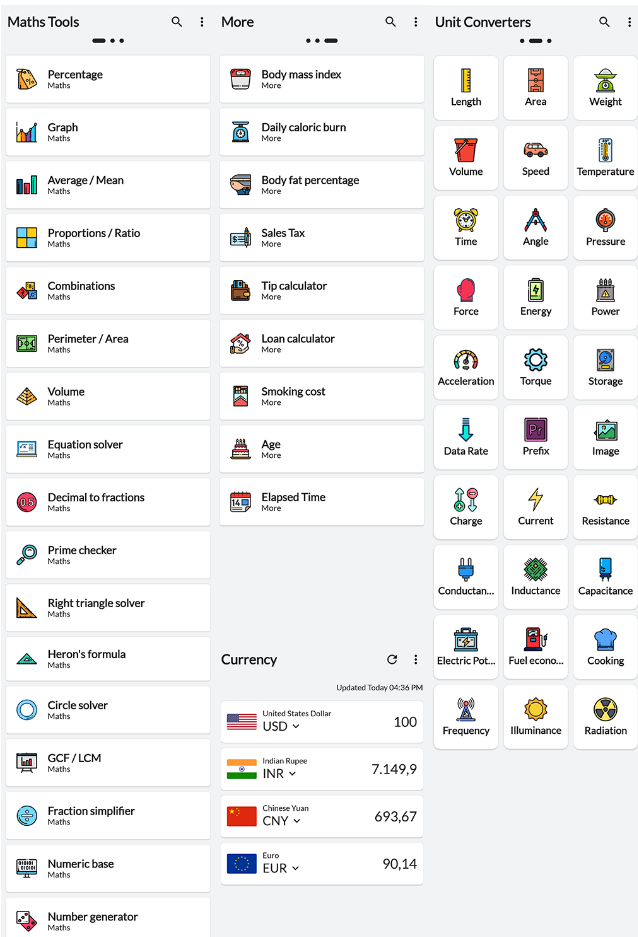

Figure 6: Screenshot of the additional functions available in the "Calculator Plus" app.

Another way to hide underlying complexity is to make calculation specific to the units used. As discussed above, conversion is an example of this, but we found a range of other unit-specific functionality. For example, a building calculator allows users to work with 'cement bags', 'tiles', or 'bricks' directly. A common example is calculation with time, where users can, for example, directly express the subtraction of two dates. With physical calculators, users would instead first need to convert dates to an integer representation to calculate with them.

The problem with these aides is that they are only available as specialized functions. There is no consistent abstraction and hence users need to pick the right function and app for their problem.

B.4 Aligning with the World

One important way to ease application of mathematics is to align it better with the world. The better users can transfer a problem they are facing into a mathematical form, the more effective they can be. For example, providing dedicated support for percentages can help users express what they see in their daily life. When a store has "30% off" signs, for example, some calculators require users to manually translate that into a multiplication of "Price × 0.7". Yet, other calculators allow for more direct translations as "Price × 70%" or even "Price − 30%". Because such formulations might not be straightforward for all users, several calculators offer special discount views, where there are dedicated fields for the starting price and the percentage reduction. Hence, in those cases, users need to only copy in values, but no longer need to formulate the target equation.

Better alignment of the calculator interface with the problem is also prominent with geometry-related functionality. For example, these commonly include illustrations of the corresponding shapes and solids. Hence, users can directly see what variable corresponds to what aspect of a shape. Going a step further, some apps allow number entry directly in the displayed shapes. For example, users can enter lengths on two sides of a triangle and then automatically get the length of the third computed.

Ultimately, while calculator apps close the gulf of execution somewhat, there remains a disconnect between themselves and the tasks. The forms and views they provide only align with or mimic the respective task, but remain distinct from them. For example, the geometric views in the apps are only iconic representations of the actual objects they are used for.